Due Fri

Review / warm up

$$\begin{bmatrix}1\\0\\0\end{bmatrix}x_1+\begin{bmatrix}3\\0\\0\end{bmatrix}x_2+\begin{bmatrix}0\\1\\0\end{bmatrix}x_3+\begin{bmatrix}2\\-1\\0\end{bmatrix}x_4+\begin{bmatrix}0\\0\\1\end{bmatrix}x_5$$

(6 pts) The augmented matrix for a linear system of equations has been reduced to reduced row echelon form. Express the solution set as a linear combination of column vectors that contain only numerical entries.

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5$

$$\left[\begin{array}{ccccc|c} 1 & 3 & 0 & 2 & 0 & 5 \\ 0 & 0 & 1 & -1 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & -2 \end{array}\right] \rightarrow \begin{aligned} x_1 + 3x_2 + 2x_4 &= 5 \\ x_3 - x_4 &= 3 \\ x_5 &= -2 \end{aligned}$$

$A\vec{x} = \vec{b}$

$$x_1 = -3x_2 - 2x_4 + 5$$
$$x_3 = x_4 + 3$$
$$x_5 = -2$$

Let $s = x_2$, $t = x_4$

$$\vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix}$$

$$\vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\\x_5\end{bmatrix} = \begin{bmatrix}-3s-2t+5\\s\\t+3\\t\\-2\end{bmatrix}$$

$$\vec{x} = \begin{bmatrix}-3\\1\\0\\0\\0\end{bmatrix}s + \begin{bmatrix}-2\\0\\1\\1\\0\end{bmatrix}t + \begin{bmatrix}5\\0\\3\\0\\-2\end{bmatrix}$$

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Then $\det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = |A|$

$= ad - bc$

This is called the arrow technique

2.1 – Determinants by Cofactor Expansion

Definition 1: If A is a square matrix, then the **minor of entry a_{ij}** is denoted by M_{ij} and is defined to be the determinant of the submatrix that remains after the ith row and jth column are deleted from A. The number $(-1)^{i+j} M_{ij}$ is denoted by C_{ij} and is called the **cofactor of entry a_{ij}**.

$A = [a_{ij}]$

a_{11} a_{12} a_{13} Matrix notation

Ex: Find all the minors and cofactors of $A = \begin{bmatrix} 2^{+} & 3^{-} & -7^{+} \\ 4^{-} & -2^{+} & -9^{-} \\ 2^{+} & 5^{-} & 6^{+} \end{bmatrix}$.

$(-1)^{i+j}$

$M_{11} = \begin{vmatrix} -2 & -9 \\ 5 & 6 \end{vmatrix}$ $= -12 + 45$

$M_{11} = 33$

$C_{11} = 33$

$M_{12} = \begin{vmatrix} 4 & -9 \\ 2 & 6 \end{vmatrix}$

$M_{12} = 42$

$C_{12} = -42$

$M_{13} = \begin{vmatrix} 4 & -2 \\ 2 & 5 \end{vmatrix}$

$M_{13} = 24$

$C_{13} = 24$

$M_{21} = \begin{vmatrix} 3 & -7 \\ 5 & 6 \end{vmatrix}$

$M_{21} = 53$

$C_{21} = -53$

$M_{22} = \begin{vmatrix} 2 & -7 \\ 2 & 6 \end{vmatrix}$

$M_{22} = 26$

$C_{22} = 26$

$M_{23} = \begin{vmatrix} 2 & 3 \\ 2 & 5 \end{vmatrix}$

$M_{23} = 4$

$C_{23} = -4$

$M_{31} = \begin{vmatrix} 3 & -7 \\ -2 & -9 \end{vmatrix}$

$M_{31} = -41$

$C_{31} = -41$

$M_{32} = \begin{vmatrix} 2 & -7 \\ 4 & -9 \end{vmatrix}$

$M_{32} = 10$

$C_{32} = -10$

$M_{33} = \begin{vmatrix} 2 & 3 \\ 4 & -2 \end{vmatrix}$

$M_{33} = -16$

$C_{33} = -16$

Definition 2: If A is an $n \times n$ matrix, then the number obtained by multiplying the entries in any row or column of A by the corresponding cofactors and adding the resulting products is called the **determinant of A**, and the sums themselves are called **cofactor expansions of A**. That is,

$$\det(A) = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj}$$

(cofactor expansion along the jth column)

$$= a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in}$$

(cofactor expansion along the ith row)

Ex: Compute the determinant of the matrix A above.

$$\begin{vmatrix} 2 & 3 & -7 \\ 4 & -2 & -9 \\ 2 & 5 & 6 \end{vmatrix} = 2(33) + 3(-42) - 7(24)$$

$$= -228$$

$$= +2\begin{vmatrix} -2 & -9 \\ 5 & 6 \end{vmatrix} - 3\begin{vmatrix} 4 & -9 \\ 2 & 6 \end{vmatrix} + (-7)\begin{vmatrix} 4 & -2 \\ 2 & 5 \end{vmatrix}$$

11. Use the arrow technique of Figure 2.1.1 to evaluate the determinant. $20 + 84 + 6 = 110$

$$\begin{vmatrix} -2 & 1 & 4 \\ 3 & 5 & -7 \\ 1 & 6 & 2 \end{vmatrix} \begin{matrix} -2 & 1 \\ 3 & 5 \\ 1 & 6 \end{matrix}$$

$-20 - 7 + 72$

$45 - 110 = -65$

only for 3x3

2x2

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} \cdots$$

$aei - afh$

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} \begin{matrix} a & b \\ d & e \\ g & h \end{matrix}$$

$-afh \quad aei$

Theorem 2.1.1 If A is an $n \times n$ matrix, then regardless of which row or column of A is chosen, the number obtained by multiplying the entries in that row or column by the corresponding cofactors and adding the resulting products is always the same.

lambda

16. Find all values of λ for which $\det(A) = 0$.

$$A = \begin{bmatrix} \lambda - 4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda - 1 \end{bmatrix}$$

$$\det(A) = 0 \Rightarrow \begin{vmatrix} \lambda-4 & 0 & 0 \\ 0 & \lambda & 2 \\ 0 & 3 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda-4)\begin{vmatrix} \lambda & 2 \\ 3 & \lambda-1 \end{vmatrix} = 0$$

$$(\lambda-4)(\lambda^2-\lambda-6) = 0$$

$$(\lambda-4)(\lambda-3)(\lambda+2) = 0$$

$$\boxed{\lambda = -2,\ 3,\ 4}$$

Theorem 2.1.2 If A is an $n \times n$ triangular matrix (upper triangular, lower triangular, or diagonal), then $\det(A)$ is the product of the entries on the main diagonal of the matrix; that is, $\det(A) = a_{11}a_{22}\cdots a_{nn}$.

31. Evaluate the determinant of the given matrix by inspection.

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 0 & 1 & -4 & 1 \\ 0 & 0 & 2 & 7 \\ 0 & 0 & 0 & 3 \end{bmatrix} = 1(1)(2)(3) = 6$$

Back to 1.10

polynomial interpolation

$$y = a_3x^3 + a_2x^2 + a_1x + a_0$$

$(2,1)$: $8a_3 + 4a_2 + 2a_1 + a_0 = 1$

$(3,5)$ $27a_3 + \cdots$ —

$(-4,7)$

$(1,2)$

$$\begin{bmatrix} 8 & 4 & 2 & 1 & 1 \\ 27 & & \text{stuff} & & \\ 256 & & \text{stuff} & & \\ 1 & & \text{stuff} & & \end{bmatrix}$$

↑ This is lousy

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$(2,1)$: $a_0 + 2a_1 + 4a_2 + 8a_3 = 1$

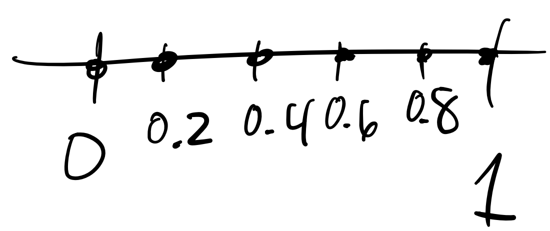
$(3,5)$ $a_0 + 3a_1 +$ stuff

$(-4,7)$ $a_0 - 4a_1$ — ——

$(1,2)$ $a_0 + a_1$ — ——

$$\begin{bmatrix} 1 & 2 & 4 & 8 & 1 \\ 1 & 3 & \text{stuff} & & \\ 1 & -4 & & & \\ 1 & 1 & & & \end{bmatrix}$$

$$\int_0^1 e^{x^2}\,dx$$



x	e^{x^2}
0	1
0.2	$e^{0.04}$

$$ax^5 + bx^4 + cx^3 + dx^2 + ex = y - 1$$

$$0.2^5 a + 0.2^4 b + \cdots = e^{0.04} - 1$$

$$\left[\begin{array}{cccc|c} 0.2^5 & 0.2^4 & 0.2^3 & \cdots & e^{0.04}-1 \\ & & & & \end{array}\right]$$